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Translated by M.D.F.

PMM U.S.S.R., Vol.53, No.6, pp.815-820, 1989
Printed in Great Britain

0021-8928/89 \$10.00+0.00

SLIP LINES AT THE CORNER OF THE INTERFACIAL BOUNDARY OF DIFFERENT MEDIA*

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A symmetric problem on the initial development of a plastic wave, simulated by two straight slip lines starting from the apex, near the corner of the interfacial boundary of different media is examined under plane strain conditions. The exact analytical solution is constructed for the Wiener-Hopf functional equation of the problem. A formula is deduced to determine the slip line length, and their slope to the interfacial boundary of the media is established.

1. We consider the problem of the initial development of the plastic zone near a corner O of the interfacial boundary of media (Fig.1) under plane strain conditions in a domain consisting of two homogeneous isotropic parts *1* and *2* with Young's moduli and Poisson's ratios E_1, ν_1 and E_2, ν_2, respectively. The problem is assumed to be symmetrical about the bisectrix of the corner. It is assumed that the plastic strains are concentrated along two straight slip lines starting from the apex, whose length is small compared with the body dimensions.

Using the "microscope principle", we arrive at a plane static symmetric problem of elasticity theory of the class N /1/ for a piecewise-homogeneous plane with interfacial boundary of the media in the form of the sides $\theta = \beta$ and $\theta = \beta - 2\alpha$ ($\alpha \in]0;\ \pi/2\ [\bigcup]\ \pi/2;\ \pi[$) of an angle containing slip lines for $\theta = 0,\ r < l$ and for $\theta = 2(\beta - \alpha),\ r < l$. An asymptotic form is realized at infinity that is the greatest solution, at infinity, of an analogous problem for a piecewise-homogeneous plane without slip lines that satisfies the stress decay condition at infinity. The latter is constructed by the method of singular solutions /1/ and is determined apart from an arbitrary constant C. This constant, that characterizes the external field strength, is considered given. It is found from the solution of the external problem.

It is required to determine the slip line length l and the angle β of their slope to the interfacial boundary of the media.

Confining ourselves to an examination of the half-plane $\beta - \alpha \leqslant \theta \leqslant \pi - \alpha + \beta$, we write the boundary conditions thus:

$$\theta = \beta,\ \langle\sigma_\theta\rangle = \langle\tau_{r\theta}\rangle = 0,\ \langle u_\theta\rangle = \langle u_r\rangle = 0 \tag{1.1}$$

$$\theta = \beta - \alpha,\ \theta = \pi - \alpha + \beta,\ \tau_{r\theta} = 0,\ u_\theta = 0$$

$$\theta = 0,\ \langle\sigma_\theta\rangle = \langle\tau_{r\theta}\rangle = 0,\ \langle u_\theta\rangle = 0 \tag{1.2}$$

$$\theta = 0,\ r < l,\ \tau_{r\theta} = \tau_1;\ \theta = 0,\ r > l,\ \langle u_r\rangle = 0$$

$$\theta = 0, \quad r \to l + 0, \quad \tau_{r\theta} \sim \frac{k_{II}}{\sqrt{2\pi(r - l)}} \tag{1.3}$$

$$\theta = 0, \quad r \to l - 0, \quad \left\langle\frac{\partial u_r}{\partial r}\right\rangle \sim -\frac{4(1 - \nu_1^2)}{E_1}\frac{k_{11}}{\sqrt{2\pi(l - r)}}$$

*Prikl. Matem. Mekhan., 53, 6, 1028-1033, 1989

$$\theta = 0,\ r \to \infty,\ \tau_{r\theta} = Cg_1 r^\lambda + o\,(1/r) \tag{1.4}$$

$$g_1 = g_1^{(1)}\lambda \sin \lambda\,(\alpha - \beta) - g_2^{(1)} \sin\,(\lambda + 2)(\alpha - \beta)$$

$$\begin{gathered}
g_1^{(1)} = (1 + \varkappa_2)\, k\lambda \sin 2\alpha \sin\,(\lambda + 2)\,\alpha \cos \lambda\,(\pi - \alpha) \times \\
\cos\,[\lambda\,(\pi - \alpha) - 2\alpha] + (1 - \varkappa_1)\,(1 + \varkappa_2)\, k \cos \lambda\alpha \sin^2 (\lambda + 2)\,\alpha \times \\
\cos \lambda\,(\pi - \alpha) \cos\,[\lambda\,(\pi - \alpha) - 2\alpha] - (1 + \varkappa_1)\, k\lambda \sin 2\alpha \times \\
\cos \lambda\alpha \sin\,(\lambda + 2)\,\alpha \cos\,(\lambda + 2)\,\alpha + (1 + \varkappa_1)(1 - \varkappa_2)\, k \times \\
\cos \lambda\alpha \sin\,(\lambda + 2)\,\alpha \cos\,(\lambda + 2)\,\alpha \cos \lambda\,(\pi - \alpha) \sin\,[\lambda\,(\pi - \alpha) - \\
2\alpha] + (k - 1)\lambda^2 \sin^2 2\alpha \cos\,(\lambda + 2)\,\alpha + [2 - (1 - \varkappa_2)k]\,\lambda \sin 2\alpha \times \\
\cos\,(\lambda + 2)\,\alpha \cos \lambda\,(\pi - \alpha) \sin\,[\lambda\,(\pi - \alpha) - 2\alpha] + (2k - 1 + \varkappa_1)\lambda \times \\
\sin 2\alpha \cos \lambda\alpha \sin\,(\lambda + 2)\,\alpha \cos\,(\lambda + 2)\,\alpha + 2[1 - \varkappa_1 - (1 - \varkappa_2)\, k] \times \\
\cos \lambda\alpha \cos\,(\lambda + 2)\,\alpha \cos \lambda\,(\pi - \alpha) \sin\,[\lambda\,(\pi - \alpha) - 2\alpha] \sin\,(\lambda + 2)\,\alpha \\
g_2^{(1)} = (1 + \varkappa_2) k\lambda^2 \sin 2\alpha \sin \lambda\alpha \cos \lambda\,(\pi - \alpha) \cos\,[\lambda\,(\pi - \alpha) - 2\alpha] + \\
(1 - \varkappa_1)\,(1 + \varkappa_2)\, k\lambda \sin \lambda\alpha \cos \lambda\alpha \sin\,(\lambda + 2)\,\alpha \cos \lambda\,(\pi - \alpha) \cos\,[\lambda\,(\pi - \\
\alpha) - 2\alpha] - (1 + \varkappa_1)\, k\lambda^2 \sin 2\alpha \sin \lambda\alpha \cos \lambda\alpha \cos\,(\lambda + 2)\alpha + \\
(1 + \varkappa_1)(1 - \varkappa_2)\, k\lambda \sin \lambda\alpha \cos \lambda\alpha \cos\,(\lambda + 2)\,\alpha \cos \lambda\,(\pi - \\
\alpha) \sin\,[\lambda\,(\pi - \alpha) - 2\alpha] + (k - 1)\,(1 - \varkappa_1 - \lambda)\,\lambda^2 \sin^2 2\alpha \cos \lambda\alpha + \\
[2 - (1 - \varkappa_2)k]\,\lambda\,(1 - \varkappa_1 + \lambda) \sin 2\alpha \cos \lambda\alpha \cos \lambda\,(\pi - \alpha) \times \\
\sin\,[\lambda\,(\pi - \alpha) - 2\alpha] + (2k - 1 + \varkappa_1)\,\lambda\,(\lambda + 1 - \varkappa_1) \sin 2\alpha \times \\
\cos^2 \lambda\alpha \sin\,(\lambda + 2)\,\alpha + 2\,[1 - \varkappa_1 - (1 - \varkappa_2)\, k](1 - \varkappa_1 + \lambda) \cos^2 \lambda\alpha \times \\
\sin\,(\lambda + 2)\,\alpha \cos \lambda\,(\pi - \alpha) \sin\,[\lambda\,(\pi - \alpha) - 2\alpha]
\end{gathered}$$

$$k = \frac{1 + \nu_2}{1 + \nu_1} R, \quad R = \frac{E_1}{E_2}, \quad \varkappa_j = 3 - 4\nu_j \quad (j = 1, 2)$$

Here σ_θ, $\tau_{r\theta}$, σ_r are the stresses, u_θ, u_r the displacements, $\langle a \rangle$ the jump in the quantity a, and $\tau_1 = \tau_{s1}$ if $C_{g1} > 0$, $\tau_1 = -\tau_{s1}$ if $C_{g1} < 0$ (τ_{s1} is the shear yield point of material I), k_{II} is the stress intensity factor at the end of the slip line to be determined, and λ is the unique root in the interval $]-1;\ 0[$ for the equation

$$\Delta\,(-\lambda - 1) = 0,\ \ \Delta\,(z) = \delta_2\delta_3 + (q_1 q_2 \sin^2 z\pi - \delta_1\delta_4 - \delta_2\delta_3)\, k + \delta_1\delta_4 k^2,\ \ \delta_1 = \sin 2z\alpha + z \sin 2\alpha$$

$$\delta_2 = \varkappa_1 \sin 2z\alpha - z \sin 2\alpha,\ \ \delta_3 = \sin 2z\,(\pi - \alpha) - z \sin 2\alpha$$
$$\delta_4 = \varkappa_2 \sin 2z\,(\pi - \alpha) + z \sin 2\alpha,\ \ q_1 = 1 + \varkappa_1,\ \ q_2 = 1 + \varkappa_2$$

Values of $-\lambda \times 10^3$ are presented in the upper part of the table for certain values of α and R ($\nu_1 = 0.333$, $\nu_2 = 0.250$).

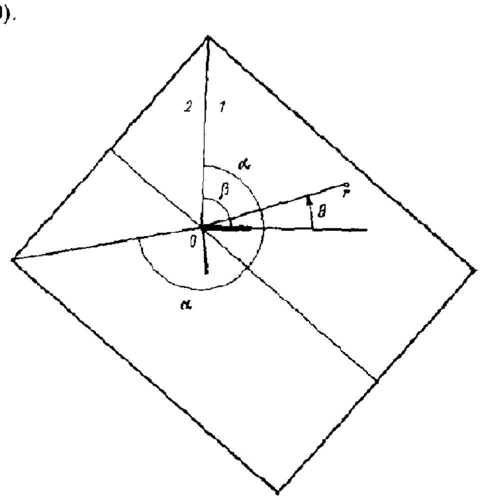

Fig.1

The solution of the problem formulated is the sum of the solutions of the following two problems. The first differs from it by the fact that we have in place of the first condition

(1.2)

$$\theta = 0,\ r < l,\ \tau_{r\theta} = \tau_1 - Cg_1 r^\lambda \tag{1.5}$$

and the stresses at infinity damp out as $o(1/r)$ (in particular, there is no first component in the expression for $\tau_{r\theta}$ in (1.4)). The second problem is the problem mentioned above for the picewise-homogeneous plane without slip lines. Since the solution of the second problem is known, it remains to construct the solution of the first.

R	α, deg=5	10	15	20	25	30	35	45
2	49	85	108	122	129	127	122	104
5	153	220	247	254	249	238	222	182
10	249	309	323	319	306	287	265	215
100	442	436	417	394	367	339	310	248
2	37	29	22	16	11	6	1	88
5	29	18	11	5	0	0	0	86
10	20	10	3	0	0	0	0	85
100	1	0	0	0	0	0	0	84

R	α, deg=65	85	105	125	145	165
2	56	11	88	132	106	48
5	96	20	168	265	248	141
10	114	23	204	330	338	235
100	131	27	244	405	463	453
2	69	51	28	7	0	0
5	68	50	26	3	0	0
10	67	49	25	0	0	0
100	66	47	23	0	0	0

2. Applying the Mellin integral transform with complex parameter p /2/ to the equilibrium equations, the strain compatibility condition, Hooke's law, and the conditions (1.1), and taking account the second condition in (1.2) and condition (1.5), we arrive at the Wiener-Hopf functional equation

$$\Phi^+(p) + \sum_{j=1}^{2} \frac{\tau_j}{p + p_j} = \operatorname{ctg} p\pi G_1(p)\,\Phi^-(p) \tag{2.1}$$

$$(-\varepsilon_1 < \operatorname{Re} p < \varepsilon_2)$$

$$\Phi^+(p) = \int_1^\infty \tau_{r\theta}(\rho l, 0)\,\rho^p\,d\rho,\quad \Phi^-(p) = \frac{E_1}{4(1-\nu_1^2)} \int_0^1 \left\langle \frac{\partial u_r}{\partial r} \right\rangle \Big|_{\substack{r=\rho l \\ \theta=0}} \rho^p\,d\rho$$

$$\tau_2 = -Cg_1 l^\lambda,\quad p_1 = 1,\quad p_2 = \lambda + 1,\quad G_1(p) = \frac{[\Delta_0(p) + \Delta_1(p)k + \Delta_2(p)k^2]\operatorname{tg} p\pi}{2\Delta(p)}$$

$$\Delta_0 = \delta_3\,[(q_1^2 - 4\delta_8)\,\delta_9 - 2\delta_6\delta_{10}]$$

$$\Delta_1 = \delta_{10}(2\delta_3\delta_6 + 2\delta_4\delta_5 + q_1q_2\delta_{11}) - \delta_9\,[\delta_3\,(q_1q_2 - 4\delta_8) - 4\delta_4\delta_7 - q_1q_2\delta_{12}],\quad \Delta_2 = -2\delta_4\,(\delta_5\delta_{10} + 2\delta_7\delta_9)$$

$$\delta_5 = \sin 2p\beta + p\sin 2\beta,\quad \delta_6 = \varkappa_1 \sin 2p\beta - p\sin 2\beta$$

$$\delta_7 = \sin^2 p\beta - p^2\sin^2\beta,\quad \delta_8 = \varkappa_1 \sin^2 p\beta + p^2 \sin^2\beta$$

$$\delta_9 = \cos 2p(\alpha - \beta) - \cos 2(\alpha - \beta),\quad \delta_{10} = \sin 2p(\alpha - \beta) + p\sin 2(\alpha - \beta)$$

$$\delta_{11} = \cos 2p(\pi - \alpha + \beta) - \cos 2(\pi - \alpha + \beta),\quad \delta_{12} = \sin 2p(\pi - \alpha + \beta) + p\sin 2(\pi - \alpha + \beta)$$

(ε_1, ε_2 and fairly small positive numbers).

The function $G_1(it)$ $(-\infty < t < \infty)$ is a real, positive, even function of t that tends to 1 as $t \to \infty$. Therefore, the index of the function $G_1(p)$ along the imaginary axis equals zero and the following factorization holds /3/

$$G_1(p) = G_1^+(p)/G_1^-(p)\ (\operatorname{Re} p = 0),\quad \exp\left[\frac{1}{2\pi i}\int_{-i\infty}^{i\infty} \frac{\ln G_1(z)}{z - p}\,dz\right] = \begin{cases} G_1^+(p), & \operatorname{Re} p < 0 \\ G_1^-(p), & \operatorname{Re} p > 0 \end{cases} \tag{2.2}$$

By using (2.2) and the factorization

$$p \operatorname{ctg} p\pi = K^+(p)\, K^-(p), \quad K^\pm(p) = \Gamma(1 \mp p)/\Gamma(^1/_2 \mp p) \tag{2.3}$$

($\Gamma(z)$ is the Gamma function), we can rewrite (2.1) thus:

$$\frac{\Phi^+(p)}{K^+(p)\, G_1^+(p)} + \frac{1}{K^+(p)\, G_1^+(p)} \sum_{j=1}^{2} \frac{\tau_j}{p + p_j} = \frac{K^-(p)\, \Phi^-(p)}{pG_1^-(p)} \quad (\operatorname{Re} p = 0) \tag{2.4}$$

Using the representation

$$\frac{\tau_j}{(p + p_j)\, K^+(p)\, G_1^+(p)} = \frac{\tau_j}{p + p_j} \left[\frac{1}{K^+(p)\, G_1^+(p)} - \frac{1}{K^+(-p_j)\, G_1^+(-p_j)} \right] + \frac{\tau_j}{(p + p_j)\, K^+(-p_j)\, G_1^+(-p_j)} \quad (\operatorname{Re} p = 0)$$

we obtain according to (2.4)

$$\frac{\Phi^+(p)}{K^+(p)\, G_1^+(p)} + \sum_{j=1}^{2} \frac{\tau_j}{p + p_j} \left[\frac{1}{K^+(p)\, G_1^+(p)} - \frac{1}{K^+(-p_j)\, G_1^+(-p_j)} \right] = \frac{K^-(p)\, \Phi^-(p)}{pG_1^-(p)} - \sum_{j=1}^{2} \frac{\tau_j}{(p + p_j)\, K^+(-p_j)\, G_1^+(-p_j)} \quad (\operatorname{Re} p = 0) \tag{2.5}$$

The function on the left-hand side of (2.5) is analytic in the half-plane $\operatorname{Re} p < 0$ while the function on its right-hand side is analytic in the half-plane $\operatorname{Re} p > 0$. On the basis of the principle of analytic continuation these functions equal the very same function that is analytic in the whole p plane.

Using (1.3) and a theorem of Abel type /4/, we find $(p \to \infty)$

$$\Phi^+(p) \sim k_{11}/\sqrt{-2pl}, \quad \Phi^-(p) \sim -k_{11}/\sqrt{2pl} \tag{2.6}$$

It follows from (2.2), (2.3), and (2.6) that the functions on the left and right-hand sides of (2.5) tend to zero as $p \to \infty$ in the half-planes $\operatorname{Re} p < 0$ and $\operatorname{Re} p > 0$ respectively. According to Liouville's theorem, a single analytic function equals zero identically in the whole plane p. Therefore, the solution of (2.1) has the form

$$\Phi^-(p) = \frac{pG_1^-(p)}{K^-(p)} \sum_{j=1}^{2} \frac{\tau_j}{(p + p_j)\, K^+(-p_j)\, G_1^+(-p_j)} \quad (\operatorname{Re} p > 0) \tag{2.7}$$

$$\Phi^+(p) = -K^+(p)\, G_1^+(p) \sum_{j=1}^{2} \frac{\tau_j}{p + p_j} \left[\frac{1}{K^+(p)\, G_1^+(p)} - \frac{1}{K^+(-p_j)\, G_1^+(-p_j)} \right] \quad (\operatorname{Re} p < 0)$$

The stresses and displacements in the problem under consideration can be determined by using (2.7) and the Mellin inversion formula.

We find from the first formula of (2.7) $(p \to \infty)$

$$\Phi^-(p) \sim p^{-1/2} \sum_{j=1}^{2} \frac{\tau_j}{K^+(-p_j)\, G_1^+(-p_j)} \tag{2.8}$$

According to (2.6) and (2.8)

$$k_{11} = \frac{\sqrt{2}\, g_1 \Gamma(\lambda + {}^3/_2)}{G_1^+(-\lambda - 1)\, \Gamma(\lambda + 2)} C l^{\lambda + 1/2} - \frac{\sqrt{\pi}}{\sqrt{2}\, G_1^+(-1)} \tau_1 \sqrt{l} \tag{2.9}$$

3. We assume no stress concentration at the end of the slip line. From (2.9) we obtain a formula to determine the length of the slip lines

$$l = D \left(\frac{|C|}{\tau_{s1}} \right)^{-1/\lambda}, \quad D = \left[\frac{2\, |g_1|\, \Gamma(\lambda + {}^3/_2)\, G_1^+(-1)}{\sqrt{\pi}\, \Gamma(\lambda + 2)\, G_1^+(-\lambda - 1)} \right]^{-1/\lambda} \tag{3.1}$$

Let us determine the direction of slip line development. According to a selection principle /5/, the value $\beta \in]0;\alpha[$, that achieves the greatest value of the velocity $V_1(\beta)$ of the dissipation energy, equal to $\tau_1 \int_0^l \langle \dot{u_r} \rangle |_{\theta=0} dr$ should be taken as the angle of its slope to the interfacial boundary of the media (the dot denotes differentiation with respect to time). Using (2.7) and (3.1), we obtain (the loading parameter C is considered to be a positive-increasing or negative-decreasing function of time)

$$V_1 = QW_1(\beta)F, \quad Q = \frac{\pi}{\lambda+2}\left[\frac{2\Gamma(\lambda+{}^3/_2)}{\sqrt{\pi}\,\Gamma(\lambda+2)}\right]^{-2/\lambda}$$

$$W_1 = |g_1|^{-2/\lambda}\frac{[G_1^+(-1)]^{-2/\lambda-2}}{[G_1^+(-\lambda-1)]^{-2/\lambda}}, \quad F = \frac{(1-\nu_1^2)\,|C|^{-2/\lambda-1}\dot{C}\,\mathrm{sign}\,C}{E_1\tau_{s1}^{-2/\lambda-2}}$$

If the slip lines are in domain *2*, then the initial problem reduces to the functional Eq.(2.1), where E_1, ν_1 in the expression for $\Phi^-(p)$ should be replaced by E_2, ν_2, τ_{s1} by τ_{s2} (τ_{s2} is the shear yield point of material *2*), the function $G_1(p)$ by the function $G_2(p)$ which differs by the fact that we have $\pi-\alpha, \varkappa_2, \varkappa_1, 1/k$ $(0<\beta<\pi-\alpha)$ respectively, in place of $\alpha, \varkappa_1, \varkappa_2, k$ and the function g_1 by the functions

$$g_2 = g_1^{(2)}\lambda\sin\lambda(\pi-\alpha-\beta) + g_2^{(2)}\sin(\lambda+2)(\pi-\alpha-\beta), \; g_1^{(2)} = ff_1, \; g_2^{(2)} = ff_2$$

$$f = (1-\varkappa_1+\lambda)\cos\lambda\alpha\sin(\lambda+2)\alpha - \lambda\sin\lambda\alpha\cos(\lambda+2)\alpha$$

$$f_1 = (k-1)\lambda\sin 2\alpha\cos[\lambda(\pi-\alpha)-2\alpha] + (2k-1+\varkappa_1)\cos\lambda\alpha \times \sin(\lambda+2)\alpha\cos[\lambda(\pi-\alpha)-2\alpha] + (1+\varkappa_1)\cos\lambda\alpha\cos(\lambda+2)\alpha \times \sin[\lambda(\pi-\alpha)-2\alpha]$$

$$f_2 = \lambda[\lambda+2-k(1-\varkappa_2+\lambda)]\sin 2\alpha\cos\lambda(\pi-\alpha) + [(1-\varkappa_1)(\lambda+2)-2k(1-\varkappa_2+\lambda)]\cos\lambda\alpha\sin(\lambda+2)\alpha\cos\lambda(\pi-\alpha) - (1+\varkappa_1)\lambda\times\cos\lambda\alpha\cos(\lambda+2)\alpha\sin\lambda(\pi-\alpha)$$

The following formula holds for the rate of energy dissipation $V_2(\beta)$:

$$V_2 = QW_2^*(\beta)F, \; W_2^*(\beta) = W_2(\beta)R(1-\nu_2^2)/(1-\nu_1^2)\,\mu^{-2/\lambda-2}, \; \mu = \tau_{s1}/\tau_{s2}.$$

Here W_2 differs from W_1 by the fact that we have g_2, G_2^+, respectively, in place of g_1, G_1^+ (G_2^+ is defined by (2.2) where G_1 should be replaced by G_2).

Let $R>1$. Investigations show that the greatest value of the function $W_2^*(\beta)$ $(0\leqslant\beta\leqslant\pi-\alpha)$ is greater than the greatest value of the function $W_1(\beta)$ $(0\leqslant\beta\leqslant\alpha)$. Consequently, the slip lines will develop in the domain *2*.

The dependence $W_2(\beta)$ is shown qualitatively in Fig.2. Graphs *1-5* refer to the cases $0<\alpha<\alpha_1, \alpha_1\leqslant\alpha<\alpha_2, \alpha_2<\alpha<\pi/2, \pi/2<\alpha<\alpha_3, \alpha_3\leqslant\alpha<\pi$ respectively ($\alpha_m = \alpha_m(R, \nu_1, \nu_2)$, $m = 1, 2, 3$).

Analysing the function $W_2(\beta)$, the following deductions can be made on the basis of the selection principle for the initial symmetrical problem.

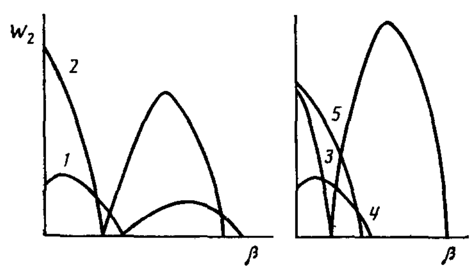

Fig.2

Let R, ν_1, ν_2 be fixed. For $0<\alpha<\alpha_1$ the slip lines develop at an angle to the interfacial boundary of the media that decreases as α increases. For $\alpha_1\leqslant\alpha<\alpha_2$ they develop along the interfacial boundary of the media. If the case $\alpha=\alpha_2$ is realized, then four slip lines start from the corner, two of which are on the interfacial boundary of the media, and two at an angle to it. For $\alpha_2<\alpha<\pi/2$ and $\pi/2<\alpha<\alpha_3$ the slip lines again make an angle with the interfacial boundary of the media that diminishes as α increases, while for $\alpha_3\leqslant\alpha<\pi$ they develop along it.

Let R grow, and ν_1, ν_2 be fixed. the domain $]0; \alpha_1[$ of values α, for which the initial deviation of the slip lines from the interfacial boundary of the media holds, diminishes. For fixed α the angle of the initial deviation of the slip lines from the interfacial boundary of the media diminishes. That value of α diminishes, starting with which the slip lines do not deviate from the interfacial boundary of the media.

Values of the slope of the slip lines to the interfacial boundary of the media are presented in degrees in the lower part of the table for certain values of α and R ($\nu_1 = 0.333$, $\nu_2 = 0.250$).

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Translated by M.D.F.

PMM U.S.S.R., Vol.53, No.6, pp. 820-821, 1989
Printed in Great Britain
0021-8928/89 $10.00+0.00

STABILITY OF MOTION OF LINEAR SYSTEMS RELATIVE TO SOME OF THE VARIABLES*

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Using the method of Lyapunov functions, we obtain the sufficient conditions for asymptotic stability of linear systems with constant coefficients, with respect to some of the variables.

Suppose we have a system of linear differential equations of perturbed motion (A is a constant $(n \times n)$ matrix):

$$\mathbf{x}^{\cdot} = A\mathbf{x}; \quad \mathbf{x} = (y_1, \ldots, y_m, z_1, \ldots, z_p) = (\mathbf{y}, \mathbf{z})$$
$$m > 0, \quad p \geqslant 0, \quad n = m + p \tag{1}$$

We consider the problem of the asymptotic y-stability of the unperturbed motion $\mathbf{x} = 0$ /1-4/.

Let B and B_ε be symmetric $(n \times n)$ matrices, and let $B^{(i)}$ $(i = 1, \ldots, 4)$ be matrix blocks of orders $m \times m$, $m \times p$, $p \times m$, $p \times p$, respectively, such that (E_m denotes the $(m \times m)$ identity matrix)

$$B = \begin{Vmatrix} B^{(1)} & B^{(2)} \\ B^{(3)} & B^{(4)} \end{Vmatrix}, \quad B_\varepsilon = \begin{Vmatrix} B^{(1)} - \varepsilon E_m & B^{(2)} \\ B^{(3)} & B^{(4)} \end{Vmatrix}$$
$$\varepsilon = \text{const} > 0$$

The quadratic form $v = v(\mathbf{x})$, $v(0) = 0$, is said to be: 1) positive semidefinite in all variables, if $v(\mathbf{x}) \geqslant 0$ for all $\|\mathbf{x}\| < \infty$ /5/; 2) y-positive semidefinite if $v(\mathbf{x}) \geqslant a(\|\mathbf{y}\|)$ for all $\|\mathbf{x}\| < \infty$ /2/, where $a(r)$ is a continuous and monotone increasing function of $r \in [0, \infty)$, $a(0) = 0$.

*Prikl. Matem. Mekhan., 53, 6, 1034-1035, 1989